

Numerical simulation of the buckling failure in rock slopes

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ABSTRACT: The buckling of slope in jointed rock is a special failure mode. In this paper, a numerical method is presented simulating the buckling failure process of rock slope. The calculation model is based on the geometrically nonlinear theory and implemented by using finite element method. The discontinuity behavior is simulated using "joint element". A calculation example is illustrated for a slope in an open pit mining.

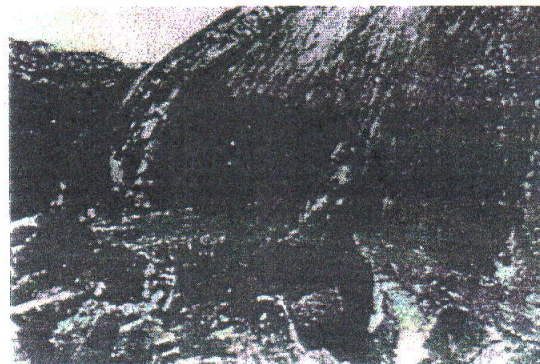
1 INTRODUCTION

It is well-known that the geological structure and strength of the rock discontinuities as well as its orientation with respect to the slope face are the essential factors to the failure of rock slope. The preexisting weak planes or discontinuities with unfavorable orientation are usually the failure surfaces of an unstable rock slope, whereas in soils it appears generally in the form of a circular arc. The pure sliding is predominately the failure mode in rock slope engineering. However, it was reported in the literature that the buckling failure of rock slope can occur if the rock mass contains one or more throughgoing discontinuities approximately parallel with the rock surface, see e.g. Fig. 1. This failure mode appears in sedimentary rocks containing slabs separated by bedding planes, and also in jointed rocks.

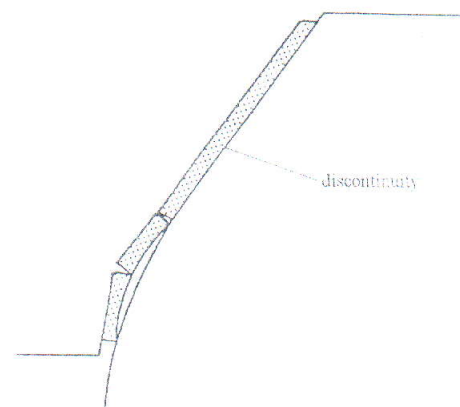
In general, the buckling failure may occur in the rock slopes, if the slope dips more steeply than the internal friction angle of the discontinuities parallel to the slope. The basic boundary conditions may be described as follows:

- a) Major discontinuity set is parallel to slope face;
- b) The spacing of discontinuity set is relatively small;
- c) The discontinuities have a low friction angle smaller than slope angle.

Kutter (1974) as well as Hoek & Bray (1977) described and discussed the buckling failure of the



a) photograph of the buckling



b) three hinge buckling model

Figure 1. A buckling failure of sandstone strata in an open pit coal mine and modeling, from Cavers (1981).

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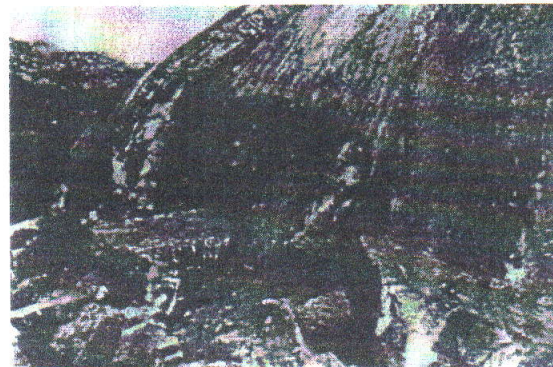
1 INTRODUCTION

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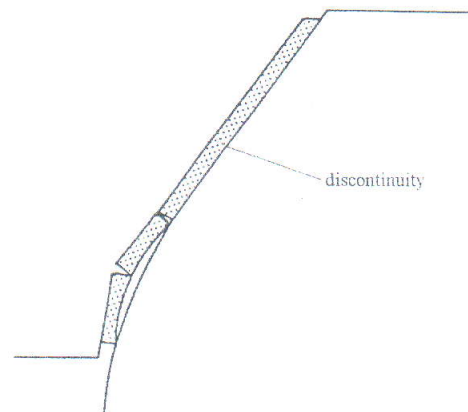
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rock slope qualitatively. For plane slope, Cavers (1981) assumed four possible failure modes of single slab lying on the slope and formulated two simple approaches to the analysis of buckling failure of rock slabs. Hu & Cruden (1992) reported the buckling of beds in the sedimentary rocks occurring on steep underdip and dip slopes in the Highwood Pass, Alberta, Canada. The observation and the analysis of the four buckling sites indicated that the bedding thickness is an important parameter determining the modes of buckling. The corresponding mechanical models were proposed for predicting the initiation of the observed buckling behavior. In these approaches, however, only the critical state at failure is referred to and the failure procedure can't be simulated.

In this paper, a numerical method is presented simulating the process of buckling failure in rock slope. A calculation example is given for the buckling failure of a rock slope in an open pit mining.

2 MODELING OF JOINTED ROCK

Jointed rock is essentially a discontinuous system. In general, there are two ways modeling its stress-strain behavior. To represent the fundamental behavior, a rock mass containing three families of discontinuities (joints) is illustrated in Fig. 2 a).

As a simplified way, the jointed rock can be approximately seen as a continuum material, see e.g. Zienkiewicz & Pande (1977), if the dimension of building is much smaller than that of joint spacing. In the replacement material, the real spacing of the discontinuities exists no longer (d_{T1} , d_{T2} , d_{T3} , compar. Fig. 2 a) and b)), and each point in this new material behaves mechanically same, whereas the orientation (striking and dip angles of the discontinuities (α_{T1} , β_{T1} , α_{T2} , β_{T2} , α_{T3} , β_{T3}) remains. This represents the deformation and strength anisotropy. The influence of discontinuities in elastic stage ($k_{N,T1}$, $k_{S,T1}$, $k_{N,T2}$, $k_{S,T2}$, $k_{N,T3}$, $k_{S,T3}$) is taken into consideration using the average values for rock mass (E_F , ν_F). The analysis using this modeling lead normally to conservative results. Separate treatment of joints becomes necessary, if the joint opening or large sliding along joints occurs. In these cases, a combined modeling seems to be computationally economical. That is, the discrete modeling is applied to the area where the joints should be individually considered, whereas the other area of the jointed rock is simulated using the homogeneous model, see Hu (1997).

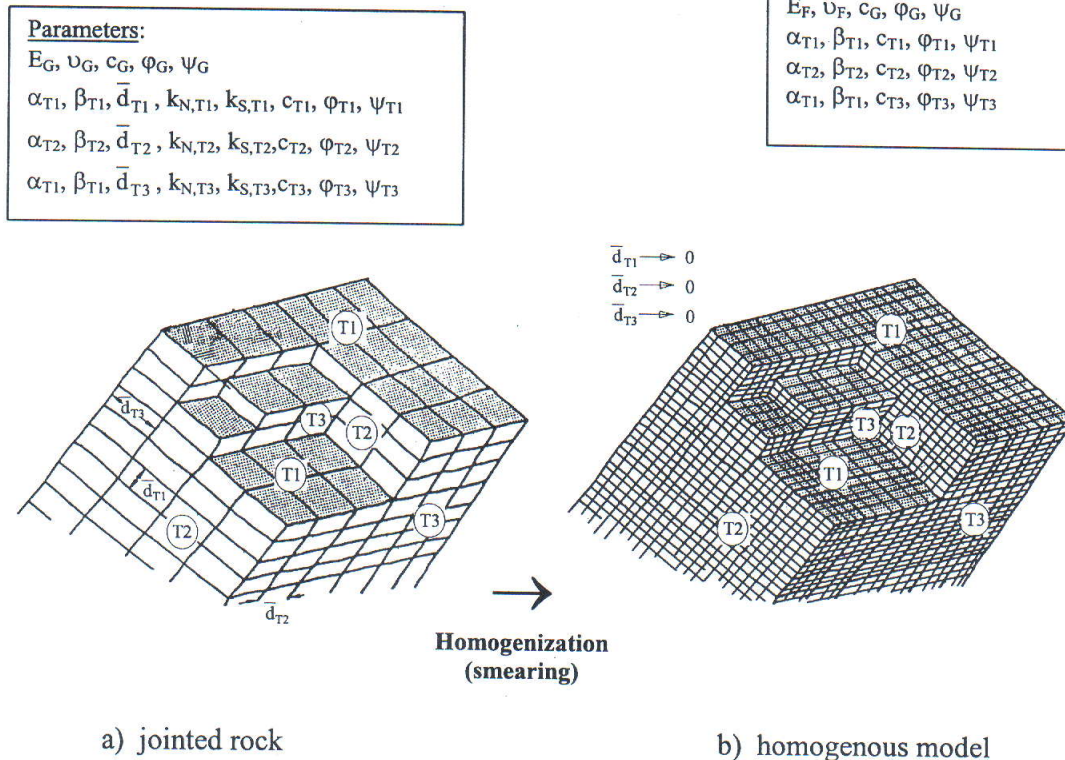


Figure 2. Modeling of jointed rocks.

Compared to this procedure, the distinct element method was specifically developed for discontinuum analysis in rock mechanics about thirty years ago, see e.g. Cundall (1988). Here, jointed rock mass is represented as an assemblage discrete blocks. All joints in rock mass are individually treated and viewed as interfaces between distinct rock blocks. The corresponding contact forces and displacements at the interfaces are determined through a iterative procedure using the principle of mechanics. For many years, however, it was seen as not-yet-proven numerical technique and has not been applied so extensively as conventional continuum analysis technique. In the recent years, the theoretical further refinement and the development of the software related to this method were made. More and more rock engineering projects are analyzed using this technique. In the near future, it may become a generally recognized tool in the analysis of rock engineering as the continuum technology.

3 FORMULATION OF THE APPLIED MODEL

For the analysis of the buckling failure of rock slope presented in this paper, the continuum model is applied with the special treatment for some key joints. In order to investigate the buckling phenomena, the geometrically nonlinear theory is used. Arising from the updated Lagrangian formulation and the elasto-viscoplastic theory, the controlling equation can be written to:

$$\int_{V(t)} \delta\{\Delta\varepsilon_t\}^T \cdot [D] \cdot \{\Delta\varepsilon_t\} dV(t) + \int_{V(t)} \delta\{\Delta\eta_t\}^T \cdot \{\sigma(t)\} dV(t) = W(t + \Delta t) - \int_{V(t)} \delta\{\Delta\varepsilon_t\}^T \cdot \{\sigma(t)\} dV(t) + \int_{V(t)} \delta\{\Delta\varepsilon_t\}^T \cdot [D] \cdot \{\Delta\varepsilon_t^{vp}\} dV(t) \quad (1)$$

- δ : variation of;
- $\{\Delta\varepsilon_t\}$: incremental linear strain vector referred to the configuration at step t;
- $[D]$: elastic matrix;
- $\{\Delta\eta_t\}$: incremental nonlinear strain vector referred to the configuration at step t;
- $\{\sigma(t)\}$: incremental linear strain vector referred to the configuration at step t;
- $\{\Delta\varepsilon_t^{vp}\}$: incremental visco-plastic strain vector referred to the configuration at step t;
- $\{\Delta u\}$: incremental displacement vector at step t+ Δt ;

- $\{f^B(t+\Delta t)\}$: body force vector at step t+ Δt ;
- $\{f^A(t+\Delta t)\}$: total surface traction vector applied at step t+ Δt ;
- $\{F^i(t+\Delta t)\}$: total concentrated load vector applied at step t+ Δt .

This equation can then be converted into a finite element formulation:

$$([K_{L,t}] + [K_{N,t}]) \cdot \{\Delta U\} = \{R_t(t+\Delta t)\} - \{F_t\} + \{\Delta F_t^{vp}\} \quad (2)$$

- $[K_{L,t}]$: linear stiffness matrix referred to the configuration at step t;
- $[K_{N,t}]$: nonlinear stiffness matrix referred to the configuration at step t;
- $\{R_t(t+\Delta t)\}$: total force applied at step t+ Δt referred to the configuration at step t;
- $\{F_t\}$: equivalent internal force vector at step t;
- $\{\Delta F_t^{vp}\}$: equivalent visco-plastic force vector at step t+ Δt referred to the configuration at step t.

Upon the finite element equation (2) a finite element program has been developed for analyzing the deformation and stability of buildings in jointed rock. In addition, "joint element" has also been implemented in this program which makes the separate treatment of some key joints possible.

4 NUMERICAL CALCULATION EXAMPLE

4.1 Details of the problem

Fig. 3 shows the cross section of an open pit coal mine as well as the planed excavation procedure. The slope is located in a geological fold and covered by two rock slabs being 0.6 m thick, respectively. The dip above the fold is 50° and below the fold 70°. The total height of the slope comes to 49 m. The discontinuities between the two slabs as well as between the underlying slope surface and the slab below have the same mechanical behavior as that of three cross joints. It is assumed that the tension strength perpendicular to the joints is zero. The geometrical and mechanical parameters of the jointed rock are given in Table 1.

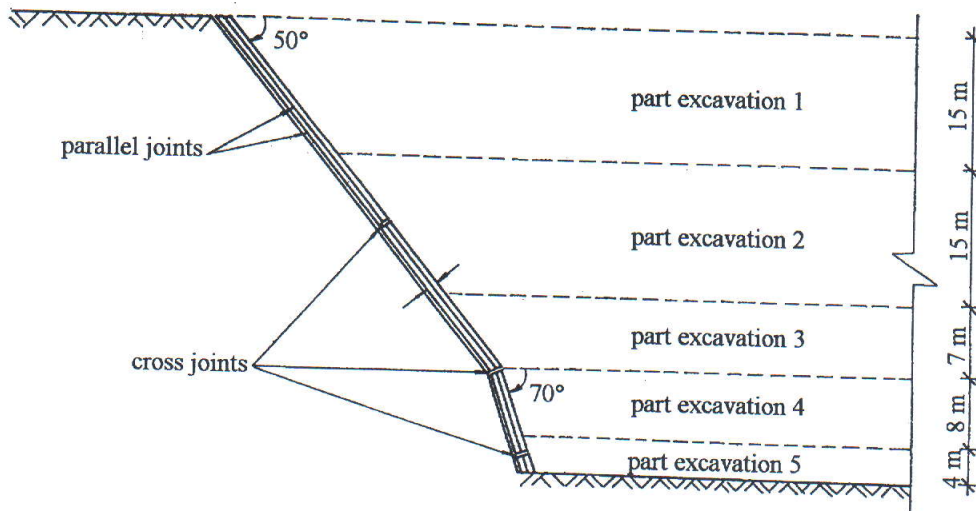


Figure 3. Details of the problem, coss section of an open pit coal mine with the rock slope.

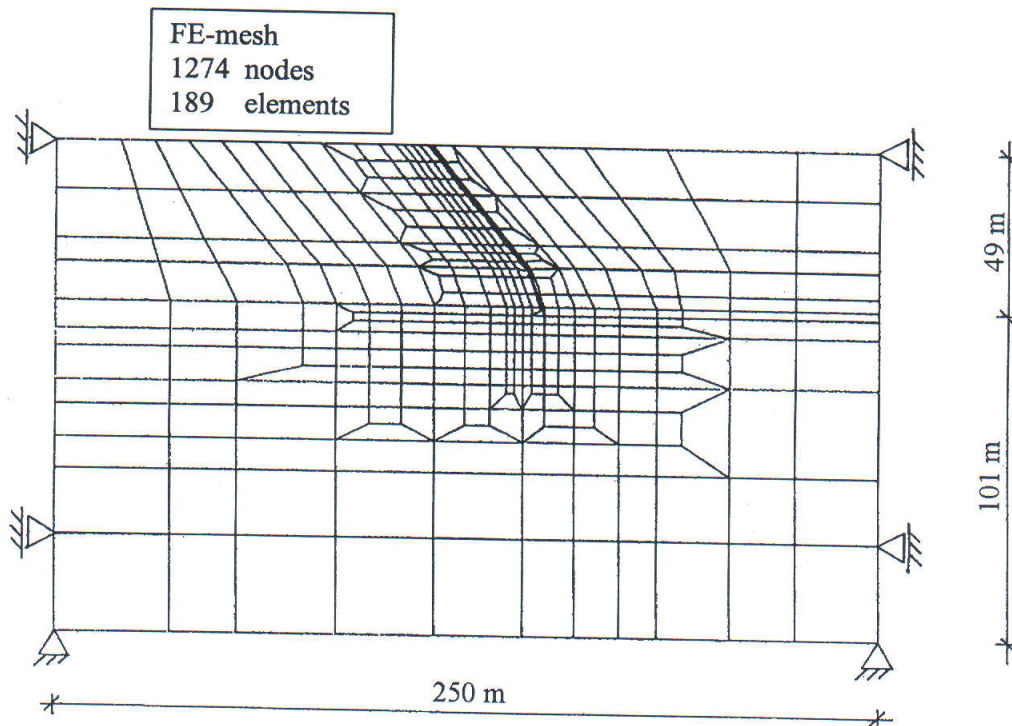


Figure 4. Computation cross section and FE-mesh.

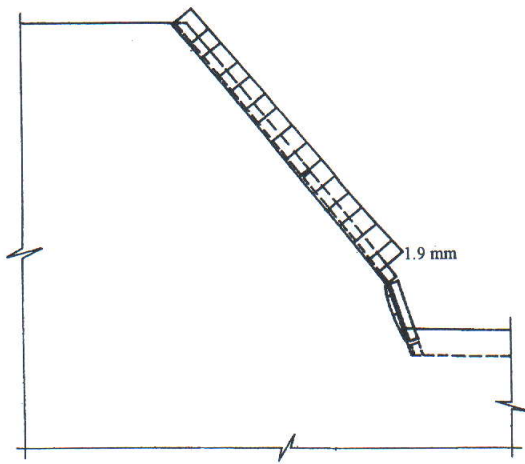
Table 1. The geometrical and mechanical parameters of the jointed rock

rock:	$\gamma = 25 \text{ kN/m}^3$; $E = 10000 \text{ MN/m}^2$; $\nu = 0.2$.
parallel joints:	$\alpha = 180^\circ$; $\beta = 50^\circ/70^\circ$; $c = 0$; $\varphi = 26^\circ$; $\psi = 12^\circ$
cross joints:	$\alpha = 180^\circ$; $\beta = 50^\circ/30^\circ/20^\circ$; $c = 0$; $\varphi = 26^\circ$; $\psi = 12^\circ$

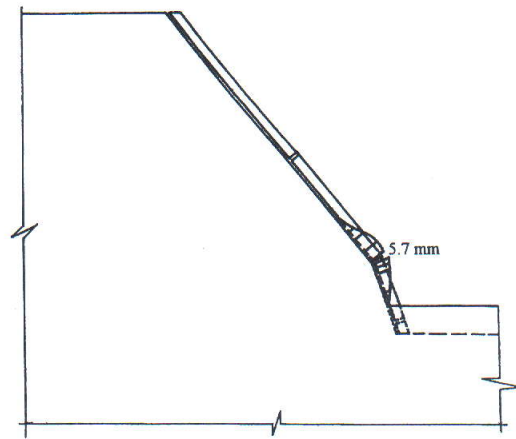
4.2 Computation cross section and FE-mesh

Fig. 4 illustrates the chosen computational cross section and FE-mesh. Apart from the area of slope surface, 8-node finite element elements were used for other area in the cross section. The rock slabs and joints on the slope surface were separately considered using finite elements and joint elements, so that the possible sliding and opening along the parallel and cross joints can be well simulated.

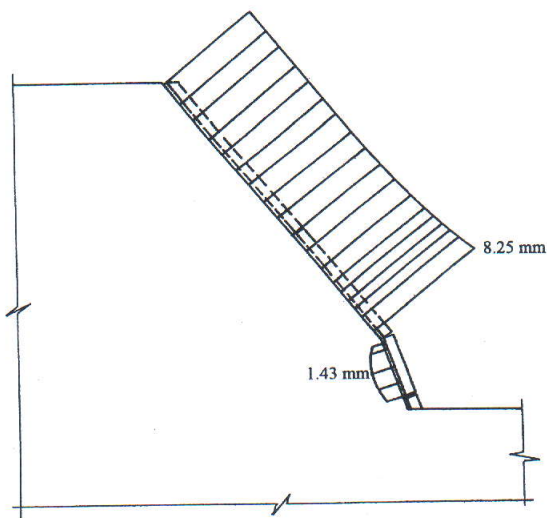
The excavation was divided into 5 part excavations in the numerical simulation, see Fig. 3. Totally 6 calculation steps are necessary. In the first step the



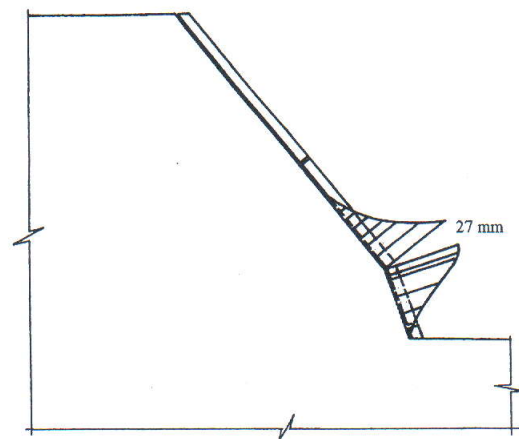
a) excavation to 45 m



a) excavation to 45 m



a) excavation to 49 m



a) excavation to 49 m

Figure 5. Relative sliding of the second slab to the slope surface.

Figure 6. Opening of the second slab to the slope surface.

primary stress state before the construction was determined. The following 5 steps simulated the 5 step excavations. The designed FE-mesh consists of 1274 nodes, 189 elements as well as 58 joint elements.

4.3 Calculation results

In Fig. 5 and 6, the sliding as well as opening of the second slab relative to the underlying slope surface are illustrated for the excavation down to 45 m and 49 m, respectively. The relative sliding of the slab part above the fold appears toward the bottom while the slab part below the fold toward the top. It comes to ca 1.9 mm at the excavation depth of 45 m and increases drastically to 8.25 mm at 49 m. At the same time, the opening of the parallel joints occurs

round the fold and increases from 5.7 mm to 27 mm at the last two stages. From the development of the relative displacement, it can be concluded that the slope is in the critical state of buckling failure. Any minor disturbance may trigger the massive slab slide. Fig. 7 gives the total displacement arising from the excavation with the reference to the primary state.

5 CONCLUSIONS

The numerical method using the geometrically nonlinear theory and the discrete modeling of joints has been applied for simulating the buckling failure of rock slope in an open pit mining. The calculation example illustrates the gradual failure process in the course of the excavation until the critical state.

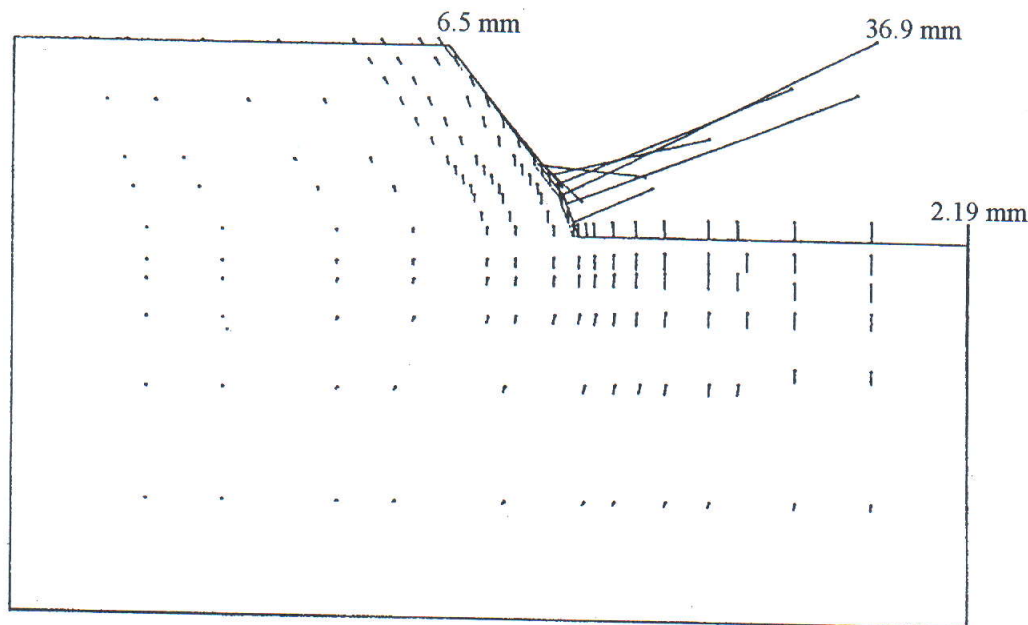


Figure 7. Total displacement with the reference to the primary state.

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